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Generalization of Common Fixed Point Theorems and Coincident Point Using Hardy-Roger Contraction and Three Self Maps in Dislocated-Quasi Metric Spaces

S. P. Birajdar*, 3N. S. Pimple, S. B. Birajdar*****
*Research Scholar, *Assistant Professor
*Department of Mathematics, N. E. S. Science College, Nanded, Maharashtra, India.
*Department of Mathematics, Rajarshi Shahu Mahavidhyalaya, (Autonomous) Latur, Maharashtra, India.
Email: shivprasadb28@gmail.com , Mob. No.: 9421500515
**, ***Assistant Professor
**, ***Department of Mathematics, Rajarshi Shahu Mahavidhyalaya, (Autonomous) Latur, Maharashtra, India.
**Email: pipnishank@gmail.com , Mob. No.: 9922717106
***Email: birajdarsujata1@gmail.com , Mob. No.: 9422023793

Abstract:

In present paper, we introduce a common fixed point theorem and coincident point with Hardy-Roger contraction and three self maps in dislocated-quasi metric space. It generalizes the some earlier theorems of fixed point theory.

Keywords: Common Fixed Point, Coincident Point, Contraction, Dislocated-Quasi Metric Space.

Introduction:

Hitzler & Seda[1] in the year 2000 proved a Fixed Point Theorem in complete dislocated metric spaces as generalization of the Banach contraction principle. Zeyada et.al.[2] in the year 2006 developed the concept of complete dislocated-quasimetric space as a generalization of dislocated metric space & the generalization of the results in such spaces. Also, Aage & Salunke[3] proved Fixed Point Theorems of continuous map in complete dislocated-quasi metric space which generalized the result of Zeyadaet.al. [2], also they proved fixed point theorems for various types of maps in complete dislocated-quasi metric space. Later Sarma et.al. [4] proved the fixed point theorems in complete dislocated-quasi metric space without considering continuity.

In present paper, we introduce the Hardy-Roger Contraction to generalize fixed point theorem in complete dislocated-quasi metric space and obtain coincidence points.

2. Complete Dislocated-Quasi Metric Space.

Definition 2.1

Let X be a nonempty set and let ρ: X × X→[0,∞) be a function satisfying the following conditions.

(I) $\rho(x,y) = \rho(y,x) = 0$ implies $x = y$.

(II) $\rho(x,y) \le \rho(x,z) + \rho(z,y)$; for all $x,y,z \in X$

Then the function ρ is called dislocated-quasi metric on X, & the pair (X, ρ) is called dislocated-quasi metric space.

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If ρ satisfies ρ (x,y)= ρ (y,x) for all x,y ε X then d is called dislocated metric & pair (X, ρ) is called dislocated metric space.

Definition 2.2

A sequence $\{x_n\}$ in dislocated-quasi metric space X is called dislocated-quasi convergent if for n ϵ N, $\lim_{n\to\infty} \rho(x_n, x) = \lim_{n\to\infty} \rho(x, x_n) = 0$

Hence x is called a dislocated-quasi limit of the sequence $\{x_n\}$

Lemma 2.1

Dislocated-quasi limit in a dislocated-quasi metric space is unique.

Lemma 2.2

Every subsequence $\{x_{n_k}\}$ of dislocated-quasi convergent to x.

Definition 2.3

A sequence $\{x_n\}$ in dislocated-quasi metric space X is called Cauchy sequence if for every $\epsilon > 0$, $\exists$ N such that ρ $(x_m, x_n) < \epsilon$ for all m, n ≥ N.

Definition 2.4

A dislocated-quasi metric space X is called complete if every Cauchy sequence in it is dislocated-quasi convergent.

Definition 2.5

Let (X, ρ) be a complete metric space and let T be a self-map on X then it is called a T Hardy-Rogers contraction if

ρ (Tx , Ty) ≤ λ_1 ρ (x, y) + λ_2 ρ (x, Tx) + λ_3 ρ (y, Ty) + λ_4 ρ (y, Tx) + λ_5 ρ (x, Ty)

for all x, y ϵ X, where $\lambda_i \geq 0$, for i = 1, 2, 3, 4, 5, such that $\lambda = \sum_{i=1}^{5} \lambda_i$ & $\lambda \epsilon [0,1)$

Definition 2.5[6]

Let the mapping f, g : X → X denotes set of coincidence points of f and g such that

C(f, g) = $\{\omega \in X: f(\omega) = g(\omega)\}$

3. Main Results

Theorem 3.1([8])

Let (X, ρ) be a complete dislocated-quasi metric space, and let S, T: X → X be a pair of self map satisfying the condition

ρ (Sx , Ty) ≤ λ_1 ρ (x , y) + λ_2 ρ (x , Sx) + λ_3 ρ (y, Ty) + λ_4 ρ (x, Ty) + λ_5 ρ (y, Sx)

for all x, y ϵ X, where $\lambda_i \geq 0$, for i = 1, 2, 3, 4, 5, such that$\lambda = \sum_{i=1}^{5} \lambda_i$ & λ ϵ [0, 1)

If $(\lambda_1 + \lambda_2 + \lambda_3 + 2\lambda_4) < 1$ then S and T have a unique common fixed point in X.

Proof:- Let us select any arbitrary point $x_0 \epsilon$ X.

We define a sequence $\{x_n\}$ in X such that S $x_{2n} = x_{2n+1}$, T$x_{2n+1} = x_{2n+2}$ for all n ϵ N.

Consider ρ (S x_{2n}, Tx_{2n+1}) = ρ (x_{2n+1}, x_{2n+2})

$\leq \lambda_1 \rho(x_{2n}, x_{2n+1}) + \lambda_2 \rho(x_{2n}, S x_{2n}) + \lambda_3 \rho(x_{2n+1}, Tx_{2n+1}) + \lambda_4 \rho(x_{2n}, Tx_{2n+1}) + \lambda_5 \rho$ $(x_{2n+1}, S x_{2n})$

$\leq \lambda_1 \rho(x_{2n}, x_{2n+1}) + \lambda_2 \rho(x_{2n}, x_{2n+1}) + \lambda_3 \rho(x_{2n+1}, x_{2n+2}) + \lambda_4 \rho(x_{2n}, x_{2n+2}) + \lambda_5 \rho$ (x_{2n+1}, x_{2n+1})

$\leq (\lambda_1+\lambda_2) \rho(x_{2n}, x_{2n+1}) + \lambda_3 \rho(x_{2n+1}, x_{2n+2}) + \lambda_4[\rho(x_{2n}, x_{2n+1}) + \rho(x_{2n+1}, x_{2n+2})]$

$\leq (\lambda_1+\lambda_2 + \lambda_4) \rho(x_{2n}, x_{2n+1}) + (\lambda_3 + \lambda_4) \rho(x_{2n+1}, x_{2n+2})$

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$\rho\,(x_{2n+1}, x_{2n+2}) \le \left(\frac{\lambda_1+\lambda_2+\lambda_4}{1-\lambda_3-\lambda_4}\right)\rho\,(x_{2n}, x_{2n+1})$

Continuing the process n times we get

$\rho\,(Sx_{2n+1}, Tx_{2n+2}) \le \left(\frac{\lambda_1+\lambda_2+\lambda_4}{1-\lambda_3-\lambda_4}\right)^{n}\rho\,(x_0, x_1)$ as n $\to\infty$ we get $\rho\,(x_n, x_{n+1}) \to 0$.

For all m, n $\in \mathbb{N}$, taking m > n, we have

$\rho\,(Sx_n, Tx_m) = \rho\,(S^n x_0, T^m x_0) \le \left(\frac{\lambda_1+\lambda_2+\lambda_4}{1-\lambda_3-\lambda_4}\right)^{n}\rho\,(Sx_0, Tx_1)$

as m, n $\to\infty$ we get $\rho\,(Sx_0, Tx_1) \to 0$. So we have $\rho\,(Sx_n, Tx_m) \to 0$ as m, n $\to\infty$.

Hence, we get a sequence $\{Tx_n\}$ which is a Cauchy sequence in a complete metric space (X, ρ) and then there exists $\omega \in$ X such that converges to T$\omega \in$ X.

Now, we prove $\omega \in$ X is a fixed point of both S and T

Suppose that f is continuous self map on X then by continuity of f, we have $\widehat{\omega} \in$ X,

$$T\widehat{\omega} = \lim_{k\to\infty} Tx_{n_k} = \lim_{k\to\infty} Tx_{n_{k-1}} = Tf\widehat{\omega}$$

But T is one to one we get $f\widehat{\omega} = \widehat{\omega}$, this shows that $\widehat{\omega}$ is a fixed point of f and uniqueness follows as $\widehat{\omega} = \omega$.

Theorem 3.2

Let (X, ρ) be a complete dislocated-quasi metric space, and S, T, ψ: X $\to$ X be three self maps satisfy the condition

ρ (Sx, Ty) $\le$ $\alpha\,\rho\,(\psi x, \psi y) + \beta\,[\rho\,(\psi x, Sx) + \rho\,(\psi x, Sy) + \rho\,(\psi x, Ty) + \rho\,(\psi y, Ty)]$

where $\alpha, \beta \ge 0$ with $\alpha + 7\beta < 1$. If S(X) $\cup$ T(X) $\subseteq \psi(X)$ which is complete dislocated-quasi subspace of X, then the three self maps have a coincidence point $\widehat{\omega}$ in X and have a unique common fixed point.

Proof: Let us select any arbitrary point $x_0 \epsilon$ X.

We define a sequence $\{y_n\}$ in X such that $y_{2n} = Sx_{2n} = \psi x_{2n+1}$,

$y_{2n+1} = Tx_{2n+1} = \psi x_{2n+2}$ for all n ϵ N $\cup$ {0}.

Consider $\rho\,(y_{2n}, y_{2n+1}) = \rho\,(Sx_{2n}, Tx_{2n+1})$

$\le\ \alpha\,\rho\,(\psi x_{2n}, \psi x_{2n+1}) + \beta\,[\rho\,(\psi x_{2n}, Sx_{2n}) + \rho\,(\psi x_{2n}, Sx_{2n+1}) + \rho\,(\psi x_{2n}, Tx_{2n+1}) +$
$\rho\,(\psi x_{2n+1}, Tx_{2n+1})\,]$

$\le\ \alpha\,\rho\,(Tx_{2n-1}, Sx_{2n}) + \beta\,[\rho\,(Tx_{2n-1}, Sx_{2n}) + \rho\,(Sx_{2n-1}, Sx_{2n+1}) + \rho\,(Tx_{2n-1}, Tx_{2n+1}) + \rho$
$(Sx_{2n}, Tx_{2n+1})\,]$

$\le\ \alpha\,\rho\,(y_{2n-1}, y_{2n}) + \beta\,[\rho\,(y_{2n-1}, y_{2n}) + \rho\,(y_{2n-1}, y_{2n+1}) + \rho\,(y_{2n-1}, y_{2n+1}) +$
$\rho\,(y_{2n}, y_{2n-1})\,]$

$\le\ \alpha\,\rho\,(y_{2n-1}, y_{2n}) + \beta\,[\rho\,(y_{2n-1}, y_{2n}) + 2\,\rho\,(y_{2n-1}, y_{2n+1}) + \rho\,(y_{2n}, y_{2n-1})\,]$

$\le\ \alpha\,\rho\,(y_{2n-1}, y_{2n}) + \beta\,[\rho\,(y_{2n-1}, y_{2n}) + 2\,\rho\,(y_{2n-1}, y_{2n}) + 2\,\rho\,(y_{2n}, y_{2n+1}) +$
$\rho\,(y_{2n}, y_{2n-1})\,]$

$\le\ \alpha\,\rho\,(y_{2n-1}, y_{2n}) + \beta\,[5\,\rho\,(y_{2n-1}, y_{2n}) + 2\,\rho\,(y_{2n}, y_{2n+1})\,]$

$\le (\alpha + 5\beta)\,\rho\,(y_{2n-1}, y_{2n}) + 2\,\beta\,\rho\,(y_{2n}, y_{2n+1})$

$\rho(y_{2n}, y_{2n+1}) \leq \left(\frac{\alpha+5\beta}{1-2\beta}\right) \rho(y_{2n}, y_{2n-1})$

Continuing the process n times we get

$\rho(y_{2n}, y_{2n+1}) \leq \left(\frac{\alpha+5\beta}{1-2\beta}\right)^n \rho(y_0, y_1)$

For all m, n $\in \mathbb{N}$, taking m > n, we have $\rho(y_n, y_m) \leq \left(\frac{\alpha+5\beta}{1-2\beta}\right)^n \rho(y_0, y_1)$

as m, n $\to \infty$ we get $\rho(y_0, y_1) \to 0$. So we have $\rho(y_n, y_m) \to 0$ as m, n $\to \infty$.

Hence, we get a sequence $\{y_n\}$ which is a Cauchy sequence in a complete metric space (X, ρ) such that $y_n = \{\psi x_n\}$ and then there exists $\omega \in \psi(X)$ such that $\{\psi x_n\}$ converges to $\omega \in$ X as n $\to \infty$. Consequently there exists $\hat{\omega} \in X$ such that $\psi(\hat{\omega}) = \omega$ then we get

$\hat{\omega} = \psi(\omega) =$ S(ω) = T(ω) if we find $\rho(\psi\hat{\omega}, S\hat{\omega})$ which tends to 0 and hence we get

$\psi(\omega) = \hat{\omega} =$ S(ω), similarly $\psi(\omega) = \hat{\omega} =$ T(ω).

This shows that $\hat{\omega}$ is a coincidence point of three self map and from above theorem 3.1 we can easily show that they have a unique fixed point.

References: -

1. Hitzler P., and Seda A. K., Dislocated Topologies, J. Engin., 51 (12/S), 3-7.
2. Zeyada F. M., Hassan G. H. and Ahmed M. A., A Generalization of a Fixed Point Theorem due to Hitzler and Seda in Dislocated Quasi Metric Spaces, The Arbian Journal for Science and Engineering, Vol. 31, No.1A, 13-22.
3. Aage C. T. and Salunke J. N., Some Results of Fixed Point Theorems in Dislocated Quasi Metric Spaces, Applied Mathematical Sciences, Vol 2, No. 59, (2008), 2941-2948.
4. M. Abbas and G, Jungck, Common Fixed Point results for non-commuting mappings without continuity in cone Metric spaces, J. Math. Anal. Appl., 341(2008), 416-420.
5. Sh. Rezapour and R. Hamlbarani, Some notes on the paper "Cone metric spaces and fixed point theorems of contractive mappings", J. Math. Anal. Appl., 345 (2) (2008), 719-724.
6. S. Radenovic, Common Fixed Points under Contractive conditions in cone Metric Spaces, Computers and Mathematics with Applications, Vol. 58, (2009), 1273-1278.
7. Sarma I. R., Rao. J. M. and Rao S. S., Fixed Point Theorems in Discolated Quasi Metric Spaces, Math. Sci. Lett., Vol. 3, No. 1, (2014), 49-52.
8. S. P. Birajdar and S. S. Zampalwad, Generalization of Some Fixed Point Theorems in POSET related Metric Space, International Journal of Emerging Technologies and Innovative Research, (JETIR) Vol. 6, Iss. 4, April (2019), 1-3.
9. S.S. Bellale and Pimple N. S., Inequality and comparison Theory for Non-linear Differential and Integral Equations, International Journal of Emerging Technologies and Innovative Research, (JETIR) Vol. 6, Iss. 4, April (2019), 281-188.
10. S. P. Birajdar and S. S. Zampalwad, Applications of Some Fixed Point Theorems in POSET Metric Spaces to Differential and Integral Equations, International Journal of Emerging Technologies and Innovative Research, (JETIR), Vol. 6, Iss. 5, May (2019), 688-691.
11. Pimple N. S. and S.S. Bellale, Inequality and Maximal-Minimal solutions for Non-linear Differential and Integral Equations, International Journal of Research in Engineering, IT, and Social Sciences (IJREISS) Vol. 9, Iss. 5, May (2019), 365-373.